$$y''-3y'+2y=3\sin 3x,\quad r^2-3r+2=0,\ (r-1)(r-2)=0, r=1,2,\quad e^x, e^{2x}\Rightarrow y_h=c_1e^x+c_2e^{2x}$$

$$(D^2-3D+2)y=3\sin 3x,\ y_p=3\frac{1}{D^2-3D+2}\sin 3x=\frac{1}{130}(-7\sin 3x+9\cos 3x)$$

$$\frac{1}{D^2-3D+2}e^{3ix}=\frac{1}{(3i)^2-3(3i)+2}e^{3ix}=\frac{1}{-9-3(3i)+2}e^{3ix}=\frac{1}{-7-9i}e^{3ix}=\frac{1}{-7-9i}\times\frac{-7+9i}{-7+9i}e^{3ix}=\frac{-7+9i}{49+81}e^{3ix}=\frac{-7+9i}{130}(\cos 3x+i\sin 3x)$$

$$=\frac{1}{130}(-7+9i)(\cos 3x+i\sin 3x)=\frac{1}{130}(-7\cos 3x-7i\sin 3x+9i\cos 3x+9i^2\sin 3x)=\frac{1}{130}\{(-7\cos 3x-9\sin 3x)+(-7\sin 3x+9\cos 3x)i\}$$

==

$$y''+y=\sin x,\ r^2+1=0,\ r^2=-1,\ r=\pm i,\ \sin x,\cos x\Rightarrow y_h=c_1\sin x+c_2\cos x$$

$$(D^2+1)y=\sin x,\ y_p=\frac{1}{D^2+1}\sin x=-\frac{1}{2}x\cos x,\quad y_p=\frac{1}{D^2+1}\cos x=\frac{1}{2}x\sin x$$

$$\frac{1}{D^2+1}e^{ix}=\frac{1}{(D+i)(D-i)}e^{ix}=\frac{1(x)}{2i(1!)}e^{ix}=(\frac{1}{i}\times\frac{i}{i})\frac{1}{2}xe^{ix}=-i\frac{1}{2}xe^{ix}=-i\frac{1}{2}x(\cos x+i\sin x)=\frac{1}{2}x\sin x+i(-\frac{1}{2}x\cos x)$$

$$a^2+b^2=a^2-i^2b^2=(a-ib)(a+ib)$$

$$y'' - y' = x^2 e^{-x}\sin x, \quad r^2 - r = 0, r = 0,1, \quad e^{0x}, e^{x} \Rightarrow y_h = c_1 + c_2 e^{x}$$

$$(D^2 - D)y = x^2 e^{-x}\sin x,\ y_p = \frac{1}{D^2 - D}(x^2 e^{-x}\sin x) = x\frac{1}{D^2 - D}(xe^{-x}\sin x) - \frac{2D-1}{(D^2 - D)^2}(xe^{-x}\sin x)$$

$$\frac{1}{D^2 - D}(xe^{-x}\sin x) = x\frac{1}{D^2 - D}(e^{-x}\sin x) - \frac{2D-1}{(D^2 - D)^2}(e^{-x}\sin x) = xe^{-x}\frac{1}{(D-1)^2 - (D-1)}\sin x - e^{-x}\frac{2(D-1)-1}{((D-1)^2 - (D-1))^2}\sin x$$

$$\frac{2D-1}{(D^2 - D)^2}(xe^{-x}\sin x) = x\frac{2D-1}{(D^2 - D)^2}(e^{-x}\sin x) - \frac{2(D^2 - D)^2 - 2(D^2 - D)(2D-1)^2}{(D^2 - D)^4}(e^{-x}\sin x)$$

$$= xe^{-x}\frac{2(D-1)-1}{((D-1)^2 - (D-1))^2}(\sin x) - e^{-x}\frac{2((D-1)^2 - (D-1))^2 - 2((D-1)^2 - (D-1))(2(D-1)-1)^2}{((D-1)^2 - (D-1))^4}(\sin x)$$

18.8.2) $$\frac{1}{D^2+1}x^2\cos x = x\frac{1}{D^2+1}(x\cos x) - \frac{2D}{(D^2+1)^2}(x\cos x)$$

$$\frac{1}{D^2+1}(x\cos x) = x\frac{1}{D^2+1}(\cos x) - \frac{2D}{(D^2+1)^2}(\cos x)$$

$$\frac{2D}{(D^2+1)^2}(x\cos x) = x\frac{2D}{(D^2+1)^2}(\cos x) + \frac{2(D^2+1)^2 - 2(D^2+1)(2D)^2}{(D^2+1)^4}\cos x$$