

$$\begin{cases} \frac{dx}{dt} = 2x + 3y \\ \frac{dy}{dt} = x + 4y \end{cases} \Rightarrow \begin{cases} (D-2)x - 3y = 0 \\ (D-2)(-x + (D-4)y = 0) \end{cases} \Rightarrow (-3 + D^2 - 6D + 8)y = 0 \Rightarrow (D^2 - 6D + 5)y = 0 \Rightarrow (D-1)(D-5)y = 0 \Rightarrow y = c_1 e^t + c_2 e^{5t}$$

$$\begin{cases} (D-4)((D-2)x - 3y = 0) \\ 3(-x + (D-4)y = 0) \end{cases} \Rightarrow x = \frac{dy}{dt} - 4y \Rightarrow x = c_1 e^t + 5c_2 e^{5t} - 4c_1 e^t - 4c_2 e^{5t} = -3c_1 e^t + c_2 e^{5t}$$

$$y(0) = c_1 + c_2 = 1, \quad x(0) = -3c_1 + c_2 = 0 \Rightarrow 4c_1 = 1 \Rightarrow c_1 = \frac{1}{4}, \quad c_2 = \frac{3}{4}$$

$$\begin{cases} \frac{dx}{dt} = 2x + 3y \\ \frac{dy}{dt} = x + 4y \end{cases} \quad x(0) = 0, y(0) = 1 \Rightarrow \begin{cases} sX(s) - 0 = 2X(s) + 3Y(s) \\ sY(s) - 1 = X(s) + 4Y(s) \end{cases} \Rightarrow \begin{cases} (s-4)((s-2)X(s) - 3Y(s) = 0) \\ 3(-X(s) + (s-4)Y(s) = 1) \end{cases} \Rightarrow (s^2 - 6s + 8 - 3)X(s) = 3$$

$$\Rightarrow (s^2 - 6s + 5)X(s) = 3 \Rightarrow X(s) = \frac{3}{(s^2 - 6s + 5)} = \frac{3}{(s-1)(s-5)} = \frac{\frac{-3}{4}}{(s-1)} + \frac{\frac{3}{4}}{(s-5)} \Rightarrow x(t) = \frac{-3}{4}e^t + \frac{3}{4}e^{5t}$$

$$\begin{cases} x' + y' + 5x + 3y = e^{-t} \\ 2x' + y' + x + y = 3 \end{cases} \begin{matrix} -(D+1) \\ (D+3) \end{matrix} \Rightarrow \begin{cases} (D+5)x + (D+3)y = e^{-t} \\ (2D+1)x + (D+1)y = 3 \end{cases} \Rightarrow (-D^2 - 6D - 5 + 2D^2 + 7D + 3)x = e^{-t} - e^{-t} + 9 \Rightarrow (D^2 + D - 2)x = 9$$

$$\Rightarrow x_h = c_1 e^t + c_2 e^{-2t} \quad x_p = \frac{1}{D^2 + D - 2} 9 = \frac{-9}{2} \Rightarrow x = c_1 e^t + c_2 e^{-2t} - \frac{9}{2}$$

$$\begin{cases} \frac{dx}{dt} = 3x - 2y + e^t \\ \frac{dy}{dt} = 5x - 3y \end{cases} \quad x(0) = y(0) = 0 \Rightarrow \begin{cases} sX(s) - 0 = 3X(s) - 2Y(s) + \frac{1}{s-1} \\ sY(s) - 0 = 5X(s) - 3Y(s) \end{cases} \begin{matrix} 5 \\ (s-3) \end{matrix} \Rightarrow \begin{cases} (s-3)X(s) + 2Y(s) = \frac{1}{s-1} \\ -5X(s) + (s+3)Y(s) = 0 \end{cases}$$

$$\Rightarrow (10 + s^2 - 9)Y(s) = \frac{5}{s-1} \Rightarrow Y(s) = \frac{5}{(s-1)(s^2+1)} = \frac{\frac{5}{2}}{s-1} + \frac{Bs+C}{s^2+1}, \quad 5 = \frac{5}{2}(s^2+1) + (Bs+C)(s-1) \Rightarrow \begin{cases} B = \frac{-5}{2} \\ C = \frac{-5}{2} \end{cases}$$

$$Y(s) = \frac{\frac{5}{2}}{s-1} + \frac{\frac{-5}{2}s + \frac{-5}{2}}{s^2+1} \Rightarrow y(t) = \frac{5}{2}e^t - \frac{5}{2}\cos t - \frac{5}{2}\sin t$$