

$$(f * g)(t) = \int_0^t f(x)g(t-x)dx \quad t-x = u$$

$$= \int_t^0 f(t-u)g(u)(-du) = \int_0^t g(u)f(t-u)du = (g * f)(t)$$

$$L\left\{\int_0^t \sin x \cos(2t-2x)dx\right\} = \frac{s}{(s^2+1)(s^2+4)}$$

$$L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\} = (\sin t) * (\cos 2t)$$

$$L\left\{\int_0^t \sin(x-t)dx\right\} = -L\left\{\int_0^t \sin(t-x)dx\right\} = -\frac{1(1)}{s(s^2+1)}$$

$$L\{1 * f\} = L\{f * 1\} = L\left\{\int_0^t f(x)dx\right\} = \frac{F(s)}{s}$$

$$Y(s) + \frac{1}{s^2} Y(s) = 3 \frac{2}{s^2 + 4} \Rightarrow \left(\frac{s^2 + 1}{s^2} \right) Y(s) = \frac{6}{s^2 + 4} \Rightarrow Y(s) = \frac{6s^2}{(s^2 + 4)(s^2 + 1)}$$

$$\Rightarrow Y(s) = \frac{\frac{-24}{-3}}{(s^2 + 4)} + \frac{\frac{-6}{3}}{(s^2 + 1)} \Rightarrow Y(s) = \frac{8 \frac{1}{2} (2)}{(s^2 + 4)} + \frac{-2}{(s^2 + 1)} \Rightarrow y(t) = 4 \sin 2t - 2 \sin t$$

$$\frac{1}{s+1} = Y(s) + 2 \frac{s}{s^2 + 1} Y(s) \Rightarrow \left(\frac{s^2 + 1 + 2s}{s^2 + 1} \right) Y(s) = \frac{1}{s+1} \Rightarrow Y(s) = \frac{s^2 + 1}{(s+1)^3}$$

$$\Rightarrow y(t) = L^{-1} \left\{ \frac{s^2 + 1}{(s+1)^3} \right\} = e^{-t} L^{-1} \left\{ \frac{(s-1)^2 + 1}{s^3} \right\} = e^{-t} L^{-1} \left\{ \frac{s^2 - 2s + 2}{s^3} \right\} = e^{-t} L^{-1} \left\{ \frac{1}{s} - \frac{2}{s^2} + \frac{2}{s^3} \right\}$$
$$= e^{-t} (1 - 2t + t^2)$$

$$L\{t^n\} = \frac{n!}{s^{n+1}} \Rightarrow L^{-1} \left\{ \frac{n!}{s^{n+1}} \right\} = t^n \Rightarrow L^{-1} \left\{ \frac{1}{s^{n+1}} \right\} = \frac{t^n}{n!}$$