

$$\mathcal{L}\{\sin 3t e^{-t}\} = \frac{3}{(s+1)^2 + 9} = \frac{3}{s^2 + 2s + 1 - 1 + 10}$$

$$\mathcal{L}\{\sin 3t\} = \frac{3}{s^2 + 9},$$

$$\mathcal{L}\left\{\frac{\cos t}{e^t}\right\} = \mathcal{L}\{e^{-t} \cos t\} = \frac{s+1}{(s+1)^2 + 1}$$

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2 + 1}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2 + 3s + \frac{9}{4} - \frac{9}{4} + 6}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{\left(s + \frac{3}{2}\right)^2 + \frac{15}{4}}\right\} = e^{-\frac{3}{2}t} \mathcal{L}^{-1}\left\{\frac{s - \frac{3}{2}}{s^2 + \frac{15}{4}}\right\} = e^{-\frac{3}{2}t} \left(\mathcal{L}^{-1}\left\{\frac{s}{s^2 + \frac{15}{4}}\right\} - \mathcal{L}^{-1}\left\{\frac{\frac{3}{2}}{s^2 + \frac{15}{4}}\right\} \right) = e^{-\frac{3}{2}t} \left(\cos\left(\sqrt{\frac{15}{4}} t\right) - \mathcal{L}^{-1}\left\{\frac{\frac{3}{2} \sqrt{\frac{4}{15}} \sqrt{\frac{15}{4}}}{s^2 + \frac{15}{4}}\right\} \right)$$

$$= e^{-\frac{3}{2}t} \left(\cos\left(\sqrt{\frac{15}{4}} t\right) - \frac{3}{\sqrt{15}} \sin\left(\sqrt{\frac{15}{4}} t\right) \right)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2 + 5s + 6}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{-2}{s+2} + \frac{3}{s+3}\right\} = -2e^{-2t} + 3e^{-3t}$$

$$\mathcal{L}\{u_1(t)t^2\} = e^{-s}\mathcal{L}\{(t+1)^2\} = e^{-s}\mathcal{L}\{t^2 + 2t + 1\} = e^{-s}\left(\frac{2!}{s^3} + 2\frac{1!}{s^2} + \frac{1}{s}\right)$$

$$\mathcal{L}^{-1}\left\{e^{-3s}\frac{s}{s^2+1}\right\} = u_3(t)\cos(t-3) = \begin{cases} 0 & t < 3 \\ \cos(t-3) & t \geq 3 \end{cases}$$

$$\mathcal{L}\{\delta_2(t)(\arctan t^2 + \sin 3t)\} = e^{-2s}(\arctan 4 + \sin 6)$$

$$\mathcal{L}\{\sin t\} = \frac{1}{s^2+1} \quad \mathcal{L}\{\sin 3t\} = \frac{1}{3} \frac{1}{\left(\frac{s}{3}\right)^2+1} = \frac{1}{3} \frac{1}{\frac{s^2+9}{9}} = \frac{3}{s^2+9}$$

$$\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$$

$$\mathcal{L}\{\cos t\} = s \frac{1}{s^2+1} - \sin(0) = \frac{s}{s^2+1}$$

$$\mathcal{L}\{f''(t)\} = s\mathcal{L}\{f'(t)\} - f'(0)$$

$$= s(s\mathcal{L}\{f(t)\} - f(0)) - f'(0) = s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3\mathcal{L}\{f(t)\} - s^2f(0) - sf'(0) - f''(0)$$