

$$L\{e^{2t}\} = \frac{1}{s-2}, \quad L\{te^{2t}\} = (-1)^1 \left(\frac{1}{s-2}\right)' = -\frac{-1}{(s-2)^2} = \frac{1}{(s-2)^2}$$

$$L^{-1}\left\{\frac{1}{s+3}\right\} = e^{-3t}, \quad L^{-1}\{\ln(s+3)\} = \frac{L^{-1}\left\{\frac{1}{s+3}\right\}}{-t} = \frac{e^{-3t}}{-t}$$

$$L^{-1}\left\{\ln \frac{(s+3)^3}{(s-1)^2}\right\} = L^{-1}\{\ln(s+3)^3 - \ln(s-1)^2\} = L^{-1}\{3\ln(s+3) - 2\ln(s-1)\}$$

$$= 3L^{-1}\{\ln(s+3)\} - 2L^{-1}\{\ln(s-1)\} = 3 \frac{L^{-1}\left\{\frac{1}{s+3}\right\}}{-t} - 2 \frac{L^{-1}\left\{\frac{1}{s-1}\right\}}{-t}$$

$$= 3 \frac{e^{-3t}}{-t} - 2 \frac{e^{-t}}{-t} = \frac{2e^{-t} - 3e^{-3t}}{t}$$

$$L^{-1}\{\text{Arc tan } s\} = \frac{L^{-1}\left\{\frac{1}{1+s^2}\right\}}{-t} = \frac{\sin t}{-t}$$

$$\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{s^2+1} ds = \text{Arc tan } s \Big|_s^\infty = \frac{\pi}{2} - \text{Arc tan } s$$

$$\int_0^\infty e^{-st} \frac{f(t)}{t} dt = \int_s^\infty F(u) du \Rightarrow \int_0^\infty \frac{f(t)}{t} dt = \int_0^\infty F(s) ds$$

$$\int_0^\infty \frac{\sin t}{t} dt = \int_0^\infty \frac{1}{s^2+1} ds = \text{Arc tan } s \Big|_0^\infty = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$\int_0^\infty \frac{e^{-2t} - e^{-4t}}{t} dt = \int_0^\infty \left(\frac{1}{s+2} - \frac{1}{s+4}\right) dt = \ln(s+2) - \ln(s+4) \Big|_{s=0}^\infty$$

$$= \ln(s+2) - \ln(s+4) \Big|_{s=0}^\infty = \ln \frac{(s+2)}{(s+4)} \Big|_{s=0}^\infty = \ln 1 - \ln \frac{1}{2} = \ln 2$$

$$\mathcal{L}\{t^5 e^{2t}\} = \frac{5!}{(s-2)^6},$$

$$\mathcal{L}\{t^5\} = \frac{5!}{s^{5+1}}$$

$$\mathcal{L}^{-1}\left\{\frac{5!}{(s-2)^6}\right\} = e^{2t} \mathcal{L}^{-1}\left\{\frac{5!}{s^6}\right\} = e^{2t} 5! \frac{t^5}{5!} = e^{2t} t^5$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad \mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$$

$$\mathcal{L}\left\{\frac{\sin 2t}{e^t}\right\} = \frac{2}{(s+1)^2+4} = \frac{2}{s^2+2s+5}$$

$$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$$