$$L\{\int_0^t \sin 3t\,dt\} = \frac{3}{s^3+9s} \qquad L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L^{-1}\{\frac{1}{s^3+9s}\} = \int_0^t \frac{1}{3}\sin 3t\,dt = -\frac{1}{9}\cos 3t\,\Big|_0^t = -\frac{1}{9}\cos 3t + \frac{1}{9} \qquad L^{-1}\{\frac{1\frac{1}{3}(3)}{s^2+9}\} = \frac{1}{3}\sin 3t$$

$$L\left\{e^{2t}\int_0^t \frac{u\sin 2u}{e^u}du\right\} = \frac{1}{(s-2)}\frac{4(s-1)}{((s-1)^2+4)^2} = \frac{4(s-1)}{(s-2)(s^2-2s+5)^2}$$

$$L\{\sin 2u\} = \frac{2}{s^2+4} \qquad L\{u\sin 2u\} = -(\frac{2}{s^2+4})' = -\frac{0-4s}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$$

$$L\left\{\frac{u\sin 2u}{e^u}\right\} = L\left\{e^{-u}u\sin 2u\right\} = \frac{4(s+1)}{((s+1)^2+4)^2}$$

$$L\left\{\int_0^t \frac{u\sin 2u}{e^u}du\right\} = \frac{1}{s}\frac{4(s+1)}{((s+1)^2+4)^2}$$

$$L\{y\} = Y(s), \quad L\{y'\} = sY(s) - y(0) = sY(s) - 2, \quad L\{y''\} = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - 2s + 1$$

$$s^2Y(s) - 2s + 1 - (sY(s) - 2) - 6Y(s) = 0 \Rightarrow (s^2 - s - 6)Y(s) = 2s - 3 \Rightarrow Y(s) = \frac{2s-3}{s^2 - s - 6}$$

$$y(t) = L^{-1}\left\{\frac{2s-3}{s^2 - s - 6}\right\} = L^{-1}\left\{\frac{2s-3}{(s-3)(s+2)}\right\} = L^{-1}\left\{\frac{\frac{3}{5}}{s-3} + \frac{\frac{-7}{-5}}{s+2}\right\} = \frac{3}{5}e^{3t} + \frac{7}{5}e^{-2t}$$

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$$L\{y\} = Y(s), \quad L\{y'\} = sY(s) - y(0) = sY(s) - 1, \quad L\{y''\} = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s - 2$$

$$-(s^2Y(s) - s - 2)' + sY(s) - 1 + 2(sY(s) - 1)' - 2Y(s) = 0 \Rightarrow -(2sY(s) + s^2Y'(s) - 1) + sY(s) - 1 + 2(Y(s) + sY'(s)) - 2Y(s) = 0$$

$$\Rightarrow (-s^2 + 2s)Y'(s) + (-2s + s + 2 - 2)Y(s) = 0 \Rightarrow (-s^2 + 2s)Y'(s) = sY(s) \Rightarrow (-s + 2)Y'(s) = Y(s) \Rightarrow \frac{Y'(s)}{Y(s)} = \frac{1}{-s+2}$$

$$\Rightarrow \ln Y(s) = -\ln(2 - s) \Rightarrow Y(s) = \frac{1}{2-s} \Rightarrow y(x) = -e^{2x}$$

$$L\{x\} = X(s), \quad L\{x'\} = sX(s) - 0, \quad L\{y\} = Y(s), \quad L\{y'\} = sY(s) - 1$$

$$\begin{cases} sX(s) + 3X(s) - 4Y(s) = \dfrac{s}{s^2+1} \\ sY(s) - 1 - 3Y(s) + 2X(s) = \dfrac{1}{s^2} \end{cases} \Rightarrow \begin{cases} (s-3)((s+3)X(s) - 4Y(s) = \dfrac{s}{s^2+1}) \\ 4(2X(s) + (s-3)Y(s) = \dfrac{1}{s^2} + 1) \end{cases} \Rightarrow (s^2 - 9 + 8)X(s) = \frac{s(s-3)}{s^2+1} + \frac{4}{s^2} + 4$$

$$\Rightarrow (s^2 - 1)X(s) = \frac{s(s-3)}{s^2+1} + \frac{4+4s^2}{s^2} \Rightarrow X(s) = \frac{1}{(s^2-1)}\left(\frac{s(s-3)}{s^2+1} + \frac{4+4s^2}{s^2}\right)$$